University of Saskatchewan
Department of Mathematics & Statistics
Mathematics 110.3

Time: 3 hours | Final Examination | 9am, December 9, 1999

CLOSED BOOK EXAMINATION - NO CALCULATORS ALLOWED

Name: ______________________ Student #: ____________ Math 110 section #________

PART I

Questions in this part will be marked right or wrong. Please carefully write your answers in the spaces provided.

[7] 1. (a) $\lim_{x\to 2} x - 4 =$ __________

(b) $\lim_{t\to -2} \dfrac{t+2}{2t^3+7t^2+7t+2} =$ __________

(c) $\lim_{s\to 0} \sqrt{2} =$ __________

(d) $\lim_{h\to 0} \dfrac{(2+h)^5-32}{h} =$ __________

(e) $\lim_{\theta\to 0} \dfrac{\sin(2\theta)}{\sin(\theta)} =$ __________

(f) $\lim_{x\to 1^+} \dfrac{x-1}{|x-1|} =$ __________

(g) $\lim_{x\to 4^-} \dfrac{x^2-2}{x^2-5x+4} =$ __________

[3] 2. (a) At what x-value(s) does the graph of $y = \dfrac{x-1}{x^2-4x+3}$ have a vertical asymptote? __________

(b) Find $\lim_{x\to\infty} \dfrac{3x^2-7x+22}{1-3x^2}$. __________

(c) Find $\lim_{x\to -\infty} \dfrac{2x+12}{\sqrt{4x^2+2x+10}}$. __________

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[20] 3. Carry out the indicated differentiations. It is not necessary to simplify your answers.

(a) If $p(x) = 1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 + \frac{1}{4}x^4$,then $p'(x) =$ ____________________

(b) If $y = \tan(3x)$, then $\dfrac{dy}{dx} =$ ____________________

(c) If $w = 3^{(t-1)}$, then $\dfrac{dw}{dt} =$ ____________________

(d) If $f(t) = \frac{1}{\sqrt{2\pi}}e^{\frac{-t^2}{2}}$, then $f'(t) =$ ____________________

(e) If $g(x) = \dfrac{1}{1+x^3}$, then $g'(x) =$ ____________________

(f) If $u = \dfrac{t^2-1}{t^2+t+1}$, then $\dfrac{du}{dt} =$

____

(g) If $f(t) = t^2(\ln t)(\sin t)$, then $f'(t) =$ ____________________

(h) If $y = \ln\left[\dfrac{x^2-4}{x^2+1}\right]$, then $\dfrac{dy}{dx} =$

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(i) If $h(s) = s^{\cos s}$, then $h'(s) =$ ____________________

(j) If $V = \dfrac{4}{3}\pi r^3$, then $\dfrac{dV}{dr} =$ ____________________

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[10] 4. (a) What is the domain of the function $\ln(1-x^2)$? __________

(b) Find an antiderivative $F(x)$ of $f(x) = x^2 - \sqrt{x}$ that satisfies $F(1) = 1$. __________

(c) Complete the following definition. A function f is called <u>decreasing</u> on an interval I if __________ whenever $x_1 < x_2$ in I.

(d) Complete the following statement: If $f'(x) = g'(x)$ for all x in an interval (a, b), then there is a constant C so that __________ for $x \in (a, b)$.

(e) Complete the following statement: If f has a local maximum or minimum at c, and if $f'(c)$ exists, then __________

PART II

Please provide carefully written answers to questions 5 through 14 in an answer booklet.

[6] 5. Use the formal definition of the derivative (that is; work from first principles) to find the slope of the tangent line to the graph of $y = 2 + x^2$ at the point $(2, 6)$. (No marks will be given for using the rules of differentiation.)

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[6] 6. An oil pipeline under a large lake starts to leak. The oil comes to the surface and forms a growing circular shaped slick with a uniform thickness of 2 cm. At a given time the slick is observed to have a radius of 100 meters and the radius is increasing at a rate of 2 meters per minute. At what rate is the oil leaking from the pipe?

[6] 7. Consider $f(x) = xe^{-x}$.

(a) What is the domain of $f(x)$? At what point(s) x is $f(x) = 0$?

(b) Identify the intervals where $f(x)$ is increasing or decreasing.

(c) Identify the intervals where $f(x)$ is concave up or down.

(d) Identify any local maxima or minima of $f(x)$.

(e) Sketch a graph of $y = xe^{-x}$.

(f) Based on your graph, what do you think $\lim_{x\to\infty} xe^{-x}$ might be?

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[6] 8. A cylindrical can without a top is required to hold 8π cm^3 of liquid. What is the smallest possible area of material that can be used in making this can? (Assume there is no wastage in constructing the can.)

[6] 9. Find the equation of the tangent line to the graph of $x^{2/3} + y^{2/3} = 5$ at the point $(-8, 1)$.

[6] 10. (a) Let $f(\theta) = \sin(2\theta)$. Find $f'(\theta)$, $f''(\theta)$, $f^{(3)}(\theta)$ and $f^{(4)}(\theta)$.

(b) What is $f^{(9)}(0)$?

[6] 11. Use one step of Newton's method to estimate the cube root of 30. That is, let $f(x) = x^3 - 30$ and estimate the root of $f(x)$ by starting with an initial guess of $x_1 = 3$ and applying one step of Newton's method. Leave your answer in fractional form.

[6] 12. Let $f(x) = \dfrac{x+2}{1+x^2}$ for $x \in [-2, 2]$.

(a) Find all points $x \in [-2, 2]$ that are critical numbers for f.

(b) What are the absolute maximum value and absolute minimum value of $f(x)$ for $x \in [-2, 2]$.

[6] 13. (a) Show that the equation $x - 1 + \sin(\frac{\pi}{2}x) = 0$ has at least one solution in the interval $[0, 1]$.

(b) How many solutions to the equation in (a) are there? Give justifications for your answers. (Do not try to calculate the value of any solutions.)

[6] 14. Let c be a constant and $f(x) = x^3 + 3cx^2 + 3x + 2$. Find those values of c for which $f(x)$ has no local maximum. Verify your claim.

The End